Institutions and Social Choice Theory

Micro Theory II class notes, Temple University

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Outline

1. Institutions in Economics

2. Social Choice Theory

Why Institutions?

- We are too accustomed to thinking in terms of markets. Markets are but one class of economic institution.
- A better view of economics is available from a more general viewpoint. There an institution is a set of enforced rules that govern the behavior of participants.

Examples

- Election rules
- Constitutions
- Auction rules (more generally, allocation rules for economic commodities)
- Taxation rules
- Inheritance rules
- Matching systems (e.g., students to schools)
- Contracts
- Computer resource allocation systems

Aside: Institutions in Development Economics

Kevin Davis of NYU argues that the various “quality of legal institutions” variables that economists use in their empirical studies of what causes economic development muddle various views of institutions. They put together legal institutions along with how well the population behaves in accordance to these institutions.

We are concerned with the basic analysis of institutions as sets of enforced rules. But we will learn something about how well we can expect institutions to shape behavior, too.

Why Social Choice

Social Choice theory studies how a group of people can decide what they collectively consider desirable. Only after we know what we want can we study how to develop institutions to let us achieve it.

Notation

- Individuals: call them agents. There are I agents.
- Set of alternatives to choose from: X.
- Each agent i has preference relation R^i on X.
- $x \; R^i \; y$ means "i considers alternative x at least as desirable as alternative y".
- P^i denotes strict preference.
- I^i denotes indifference.

Assumptions on Preferences

We assume the two properties below hold for each i. With these properties satisfied, a preference relation is called a preference ordering.

- Completeness: For each $x \in X$ and each $y \in X$, either $x \; R^i \; y$ or $y \; R^i \; x$. (One must always be able to make up one's mind.)
- Transitivity: For every $x, y, z \in X$, if $x \; R^i \; y$ and $y \; R^i \; z$, then $x \; R^i \; z$. (Obvious muddleheadedness is not allowed.)

How to Express Social Choice: I

Aggregate all preference orderings into one, the social preference ordering. This is the job of a social welfare functional. A SWF maps a profile of individual preference orderings $\rho = (R^1, \ldots, R^I)$ into a social preference ordering $R(\rho)$.

Example of a SWF: The Borda Count

The Borda count is weighted voting of a kind. Each agent's ordering is used to assign a score to each alternative. The highest ranked score gets assigned one point, the second highest two, and so on. Ties are broken by averaging the ranking. If three alternatives tie for positions 10, 11, and 12, then they each are given a score of 11.

Once we have the scores from each agent, we add for each alternative all the scores and rank alternatives so that the one with the least score is best, and so on. This creates a social welfare functional. It works no matter what the alternatives and the individual orderings are.

Would-Be Example of a SWF: The Condorcet Method

The Marquis de Condorcet came up with this to extend majority rule to situations with more than two alternatives. He also found a counterexample that shows that his method does not always produce a social ranking. Hence the famous Condorcet Paradox.

Suppose there are three alternatives, $X = \{x, y, z\}$, and three agents. The preferences of the three agents are as follows:

$$x \; P^1 \; y \; P^1 \; z.$$
$$z \; P^2 \; x \; P^2 \; y.$$
$$y \; P^3 \; z \; P^3 \; x.$$

You can see the problem here easily.

Desired Properties of SWFs

We can imagine all sorts of SWFs. Many of them are pretty weird. Or a SWF can be too inequitable (for instance, the dictatorial one, which selects a particular agent and uses this agent's preference ordering as the social preference ordering, no matter what the preference orderings of all other agents are). So we impose some desirable properties and study what SWFs satisfy them.

Pareto Property

Careful, although related, this is not the same as Pareto efficiency.

Definition

A SWF R satisfies the Pareto Property if, for every $x, y \in X$ and for each $\rho = (R^1, \ldots, R^I)$, if for each i we have $x \; P^i \; y$, then $x \; P(\rho) \; y$.

If each and every agent strictly ranks x higher than y, then so must the social preference ordering.

The Borda Count has the Pareto Property.

Pairwise Independence

This used to be called Independence of Irrelevant Alternatives.

It says what happens to a third alternative's ranking should not matter for the ranking of any two alternatives, as long as they are still ranked as before.

Definition

The social welfare functional $R : \mathcal{S} \to \mathcal{R}$ has the Pairwise Independence Property if, for all pairs x, y in X, and for all preference profiles $\rho = (R^1, \ldots, R^I) \in \mathcal{S}$ and $\overline{\rho} = (\overline{R}^1, \ldots, \overline{R}^I) \in \mathcal{S}$ such that for each $i \in \mathcal{I}$ $x\ R^i\ y$ if and only if $x\ \overline{R}^i\ y$ and $y\ R^i\ x$ if and only if $y\ \overline{R}^i\ x$, we have that $x\ R(\rho)\ y$ if and only if $x\ R(\overline{\rho})\ y$ and $y\ R(\rho)\ x$ if and only if $y\ R(\overline{\rho})\ x$.

The Borda Count does not satisfy Pairwise Independence.

Dictatorial SWF

This one is an undesirable property. It says that one person's preference ordering automatically becomes the ordering of society.

Definition

A social welfare functional R is Dictatorial if there is an agent $d \in \mathcal{I}$ such that, for any x, y in X and any profile $\rho = (R^1, \ldots, R^I)$ in $\mathcal{S}$, we have that $x\ P(\rho)\ y$ if and only if $x\ P^d\ y$.

The Borda Count is clearly not dictatorial.

What We Would Love to Have

Ideally, some method to aggregate the preference orderings of all agents into a societal ordering that would accept any ordering from anyone as input, would satisfy the Pareto Property, and would satisfy Pairwise Independence. Oh, and it better not be dictatorial. Does such a SWF exist?

Sad News

Notation: $\mathcal{R}^I$ is the set of all profiles of preference orderings, and $\mathcal{P}^I$ is the set of all profiles of strict preference orderings (those that do not admit indifference between any two alternatives).

Theorem (Arrow's Impossibility Theorem)

Assume that there are at least three alternatives and that the domain of admissible individual preference profiles, denoted $\mathcal{S}$, is either $\mathcal{S} = \mathcal{R}^I$ or $\mathcal{S} = \mathcal{P}^I$. Then every social welfare functional that satisfies the Pareto property and pairwise independence is dictatorial.

This wide-ranging, dispiriting, field-founding result comes to us from Kenneth Arrow. It was the crux of his doctoral dissertation! You should write a dissertation as world-changing as he did.

Hoping for Better News

OK, trying to make a complete social ordering by mashing together individual orderings is an ambitious undertaking. Can we achieve more by being more modest? What happens if we restrict our goal to finding a top-ranked alternative for society, rather than an entire ordering of the alternatives by society?

Social Choice Functions

Definition

Let $\mathcal{S} \subseteq \mathcal{R}^I$ be given. A social choice function is a function $f : \mathcal{S} \to X$ that assigns to every profile of preference orderings $\rho = (R^1, \ldots, R^I)$ in $\mathcal{S}$ an alternative $f(\rho)$ in X.

SCF Properties: Pareto Good or Bad?

What are some good properties for social choice functions? The first one is definitely the Pareto property (ask any economist but Amartya Sen about the virtues of the Pareto criterion). OK, there was a bit of irony here. I agree with Sen that this criterion is overrated, but it still plays a huge role in the discipline and cannot be ignored.

SCF Pareto Property

We need to adapt the Pareto property so that it applies to social choice functions, as opposed to social welfare functionals. Here is our chosen version:

Definition

The social choice function $f : \mathcal{S} \to X$ has the Pareto property if, for all x in X and for all $\rho = (R^1, \ldots, R^I) \in \mathcal{S}$, if for every i we have that for all $y \in X$, $x\ R^i\ y$, then $f(\rho) = x$.

This says that if everybody considers an alternative x at least as good as any other alternative, then the social choice function picks x.

No Gaming the System!

Here's a good property, that comes from the desire to avoid liars manipulating social choice. Whoever would run the social choice function a society wants to use would not know agents' preferences, which opens the system to manipulation.

Definition

The social choice function $f : \mathcal{S} \to X$ is strategy-proof if for all i, all $\rho = (R^1, \ldots, R^i, \ldots, R^I) \in \mathcal{S}$, and all $\overline{R}^i \in \mathcal{R}$ such that $(\overline{R}^i, R^{-i})$ is in $\mathcal{S}$, $f(\rho)\ R^i\ f(\overline{R}^i, R^{-i})$.

Nonimposition

This property ensures that no alternative is ruled out from being chosen before the members of the society have expressed their preferences.

Definition

The social choice function $f : \mathcal{S} \rightarrow X$ satisfies nonimposition if for each x in X there exists $\rho = (R^1, \ldots, R^I) \in \mathcal{S}$ such that $f(\rho) = x$.

Dictatorial SCF

The dictatorial property is easy to translate to social choice functions, and just as undesirable as it was for social welfare functionals.

Definition

The social choice function f is dictatorial if there is an agent d such that, for every profile $\rho = (R^1, \ldots, R^I)$ in $\mathcal{S}$, $f(\rho) = x$ if and only if x is at the top of d's preference ranking.

More Bad News

Courtesy of Allan Gibbard and Mark Satterthwaite, working independently of each other:

Theorem (Gibbard-Satterthwaite Theorem)

Assume that the set of social alternatives X is finite and has at least three elements, that $\mathcal{S} = \mathcal{P}^I$, and that the social choice function $f : \mathcal{S} \to X$ satisfies nonimposition. Then f is strategy-proof if and only if it is dictatorial.

Where Do We Go From Here?

The social choice perspective is very general. Perhaps this is why we got such dispiriting results. We have various ways of restricting the domain of preferences, and we might get encouraging results—indeed, we will see some positive results in the rest of the course, for domains restricted to have only private goods, or private and public goods, for example, or domains with quasilinear preferences.

Social Choice Theory: Too Static?

Is social choice theory too static? Do not forget that we were very general in specifying X, the set of alternatives. Nothing prevents us from thinking of alternatives as time paths of relevant economic variables. Note: we have seen results that assume that X is finite, but plenty of similar results exist where X is infinite.